SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2006
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK #2

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 85

- Attempt questions 1 – 6
- Hand up in 3 separate booklets clearly labelled Section A, Section B and Section C

Examiner: *A. Fuller*

SECTION A

Question 1 (13 marks)

(a) Convert 80° to radians in exact form. [1]

(b) Convert $\frac{17\pi}{12}$ radians to degrees. [1]

(c) Differentiate the following:

(i) $4x^2+5$

(ii) $(2x^3-5)^4$ [2]

(iii) $\frac{3x+1}{2x-1}$ [2]

(iv) $x\sqrt{1-2x}$ [2]

(d) Find a primitive of:

(i) $\frac{2}{x^4}$

(ii) $\sqrt{x}$ [1]

(e) Find $f''(2)$ if $f(x)=x^5$ [2]

Question 2 (15 marks)

(a) $A(-1,5)$, $B(2,1)$ and $C(4,k)$ are collinear. Find the value of k. [3]

(b) Find $\int \frac{x^2+1}{x^2}dx$ [2]

(c) Evaluate $\int_{-1}^{1}(x-1)(x+1)dx$ [3]

(d) A die is tossed twice. The sum of the numbers which appear on the upmost face of the die is calculated. Using a table or otherwise:

(i) Find the probability that the sum is greater than 8. [2]

(ii) It is known that a 4 appears on the die at least once in the two throws. Find the probability that the sum is greater than 8. [2]

(e) The vertices of a triangle are $A(1,3)$, $B(8,2)$ and $C(4,-1)$.

(i) Find the coordinates of D and E, the midpoints of AC and AB respectively.

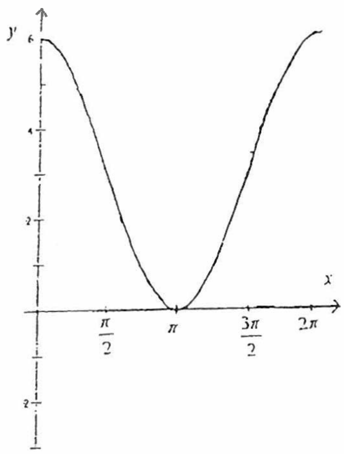
(ii) Hence, show that DE is parallel to CB. [2]

SECTION B

Question 3 (14 marks)

(a) If $y = (x^2+4)(x-3)$, solve $\dfrac{dy}{dx} = 4$. [3]

(b) The diagram below is the graph of $y = 3 + 3\cos x$



(i) Copy this graph onto your answer sheet.

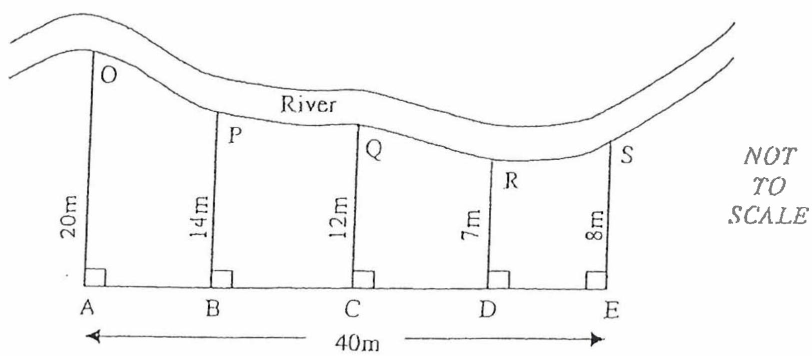
(ii) State the amplitude and period of $y = 3\sin 2x$. [1]

(iii) On the same graph as (i), sketch $y = 3\sin 2x$ for $0 \le x \le 2\pi$. [2]

(iv) How many solutions are there to the equation $3 + 3\cos x = 3\sin 2x$? [1]

(c) A class of 30 students contains 18 students who play cricket, 13 who play tennis and 5 who play both cricket and tennis. If one student is chosen at random find the probability that this student plays neither cricket nor tennis. [2]

(d) The diagram below shows a paddock with one side bounded by a river. AE is a boundary fence 40 metres in length. AO, BP, CQ, DR, ES are offsets measured from the fence to the river with lengths as shown. AB = BC = CD = DE. [3]



Use Simpson's rule with 5 function values as shown on the diagram to approximate the area of the paddock.

(e) The gradient function of a curve is $3x^2 - 1$ and the curve passes through the point $(4,1)$. Find the equation of the curve. [2]

Question 4 (13 marks)

(a) A girl has 5 tickets in a raffle where 100 tickets are sold.

First prize is drawn discarded and then the second prize is drawn.

Find the probability that she wins:

(i) first prize [1]

(ii) second prize [2]

(b) Consider the curve $y = x^4 - 4x^3$

(i) Find the coordinates of the stationary points. [2]

(ii) Determine the nature of these stationary points. [2]

(iii) For what values of x is the curve concave up? [1]

(iv) For what values of x is the curve decreasing? [1]

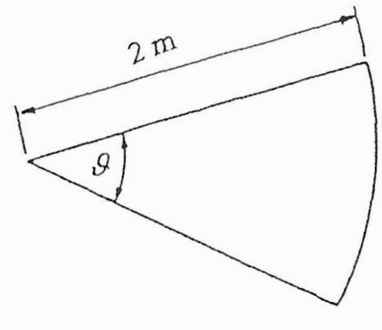
(v) Hence sketch the curve. [2]

(c) If the probability that an event E occurs is $\frac{1}{x}$, express the probability that E does not occur as a single fraction. [2]

SECTION C

Question 5 (15 marks)

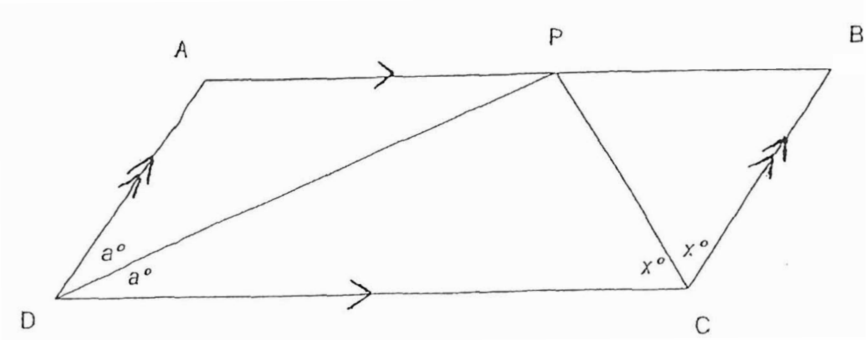
(a) A flower bed is made in the shape of a minor sector with angle ϑ and radius 2 metres.



(i) If the area of the flower bed is $1 \cdot 6m^2$, find the angle ϑ to the nearest minute. [2]

(ii) Find the perimeter of the flower bed to the nearest cm. [2]

(b) ABCD is a parallelogram. The bisectors of angles ADC and BCD meet at P on the side AB. Prove that:

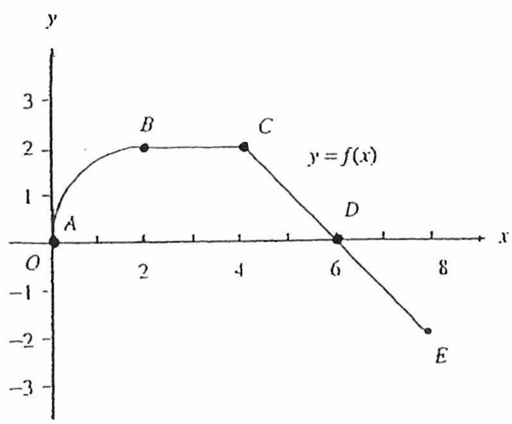


(i) $\angle$ DPC is a right angle. [2]

(ii) Δ ADP is isosceles. [2]

(iii) AB = 2BC [2]

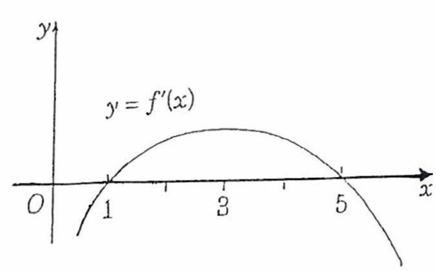
The graph below of the function f consists of a quarter circle AB and two line segments BC and CE.



(i) Evaluate $\int_0^8 f(x)dx$. [2]

(i) For what value(s) of x satisfying $0 < x < 8$ is the function f not differentiable?

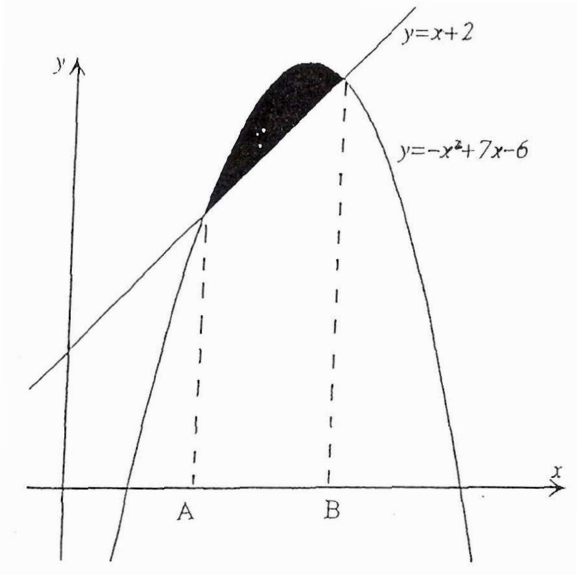
(d) The diagram below shows the graph of the gradient function of a curve. For what value(s) of x does $f(x)$ have a local maximum? Justify your answer. [2]



Question 6 (15 marks)

(a) The diagram below shows the graphs of the functions

$y = -x^2 + 7x - 6$ and $y = x + 2$.

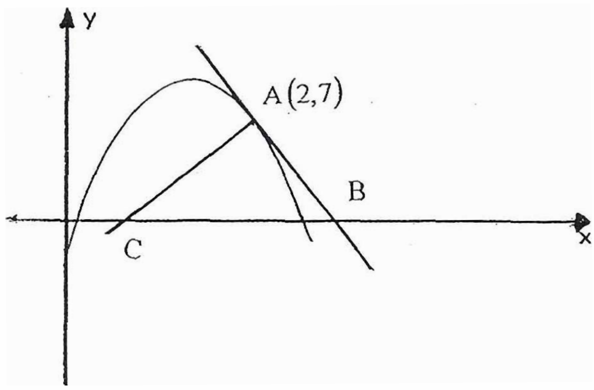


(i) Show that the value of A and B is 2 and 4 respectively. [2]

(ii) Calculate the area of the shaded region. [2]

(iii) Write down a pair of inequalities that specify the shaded region. [1]

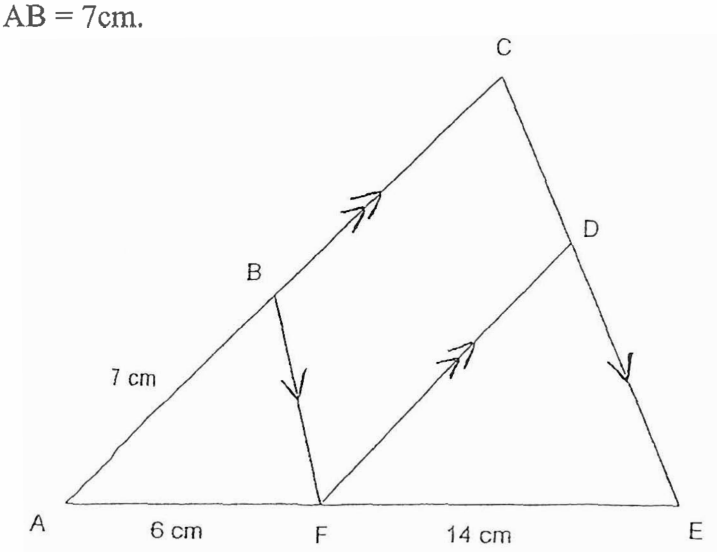
(b)



(i) Find the equation of the tangent and the equation of the normal to the curve $y=-2x^2+6x+3$ at the point $A(2,7)$. [4]

(ii) The tangent cuts the x axis at B. The normal cuts the x axis at C as shown in the diagram. Find the values of B and C. Hence, find the area of ΔABC. [2]

(c) In the diagram below AC is parallel to FD and BF is parallel to CE. B lies on AC, D lies on CE and F lies on AE. AF = 6cm, FE = 14cm and AB = 7cm.



(i) Find BC. [2]

(ii) Find the ratio of BF to DE. [2]

END OF TEST

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

Q11

2006 2U TASK 2

(a) $80 \times \frac{\pi}{180} = \frac{4\pi}{9}^{c}$

(b) $\frac{17\pi}{12} = 255^\circ$

(c) i) $\frac{d}{dx}(4x^2+5) = 8x$

ii) $\frac{d}{dx}\left((2x^3-5)^4\right) = 4 \times 6x^2 \times (2x^3-5)^3$

$= 24x^2(2x^3-5)^3$

iii) $\frac{d}{dx}\left(\frac{3x+1}{2x-1}\right) = \frac{vu' - uv'}{v^2}$

$= \frac{(2x-1)(3) - (3x+1)(2)}{(2x-1)^2}$

$= \frac{\cancel{6x} - 3 - \cancel{6x} - 2}{(2x-1)^2}$

$= \frac{-5}{(2x-1)^2}$

Q1 (c) (iv).

$$\frac{d}{dx}\left(x\sqrt{1-2x}\right) = vu' + uv'.$$

$$= \sqrt{1-2x} + x\left(\tfrac{1}{2}(1-2x)^{-\frac{1}{2}}(-2)\right)$$

$$= \sqrt{1-2x} - \frac{x}{\sqrt{1-2x}}$$

$$= \frac{1-2x-x}{\sqrt{1-2x}}$$

$$= \frac{1-3x}{\sqrt{1-2x}}.$$

d) i) $\displaystyle\int \frac{2}{x^4}\, dx = \int 2x^{-4}\, dx.$

$$= \frac{2}{-3}x^{-3} + C.$$

$$= -\frac{2}{3x^3} + C.$$

Q1) d) ii) $\int \sqrt{x}\, dx = \int x^{\frac{1}{2}}\, dx$

$$= \frac{2}{3}x^{3/2} + C$$

e) $f(x) = x^5$

$f'(x) = \dfrac{x^6}{6}$

$f''(x) = \dfrac{x^7}{7 \times 6}$

$= \dfrac{x^7}{42}$

$f''(2) = \dfrac{2^7}{42}$

$\doteq 3.05$ (2 dp)

$f(x) = x^5$

$f'(x) = 5x^4$

$f''(x) = 20x^3$

$f''(2) = 20(2)^3$

$= 160$

QUESTION (2)

a) $A(-1,5)$ $B(2,1)$.

Two pt.

$\cancel{m_1 = \frac{5-1}{-1-2} = \frac{4}{-3}}$.

$$\frac{y-1}{x-2} = \frac{5-1}{-1-2}.$$

$$-3(y-1) = 4(x-2)$$

$$-3y+3 = 4x-8$$

$$\cancel{4x=3y} \quad y = \frac{4x-11}{-3}$$

$C(4,k)$

$$y = \frac{4(4)-11}{-3}$$

$$= \cancel{\frac{-3}{-3}} \quad \frac{+5}{-3} = \boxed{\frac{5}{3}}$$

$$= 1.$$

$\therefore$ K is 1 and $C(4, \frac{-5}{3})$.

Question 2.

b). $\int \frac{x^2+1}{x^2} . dx.$

$= \int 1 + \frac{1}{x^2} . dx.$

$= \int 1 + x^{-2} . dx.$

$= x + \frac{x^{-1}}{-1} . dx.$

$= x - \frac{1}{x}. + C.$

c). $\int_{-1}^{1} (x-1)(x+1) . dx.$

$= \int_{-1}^{1} x^2 - 1 \, dx.$

$= \left[\frac{x^3}{3} - x \right]_{-1}^{1}$

$= \left[\frac{1}{3} - 1 \right] - \left[\frac{-1}{3} + 1 \right].$

$= \frac{-2}{3} - \frac{2}{3}.$

$= -\frac{4}{3} = -1\frac{1}{3}$

QUESTION (2) (d).

i)

	1	2	3	4	5	6.
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11.
6	7	8	9	10	11	12.

$P(\text{sum} > 8) = \frac{10}{36} = \frac{5}{18}.$

$P(1,4 \text{ \& sum} > 8) = \cancel{\frac{4}{36}} \quad \frac{4}{11}$

(e) $A(1,3)$ $B(8,2)$ $C(4,-1)$.

Midpoint $AB = E = \left(\frac{9}{2}, \frac{5}{2}\right)$

Midpoint $AC = D = \left(\frac{4+1}{2}, \frac{3-1}{2}\right)$

$= \left(\frac{5}{2}, 1\right).$

~~DE~~ grad $DE = \frac{5/2 - 1}{9/2 - 5/2} = \frac{3/2}{2} = \frac{3}{4}.$

grad $CB = \frac{2 - -1}{8 - 4} = \frac{3}{4}.$

So $CB \parallel DE$.

Question 3

a) If $y = (x^2 + 4)(x - 3)$, solve $dy/dx = 4$.

$y = x^3 - 3x^2 + 4x - 12$

$dy/dx = 3x^2 - 6x + 4$

$\therefore\ 4 = 3x^2 - 6x + 4$

$0 = 3x^2 - 6x$

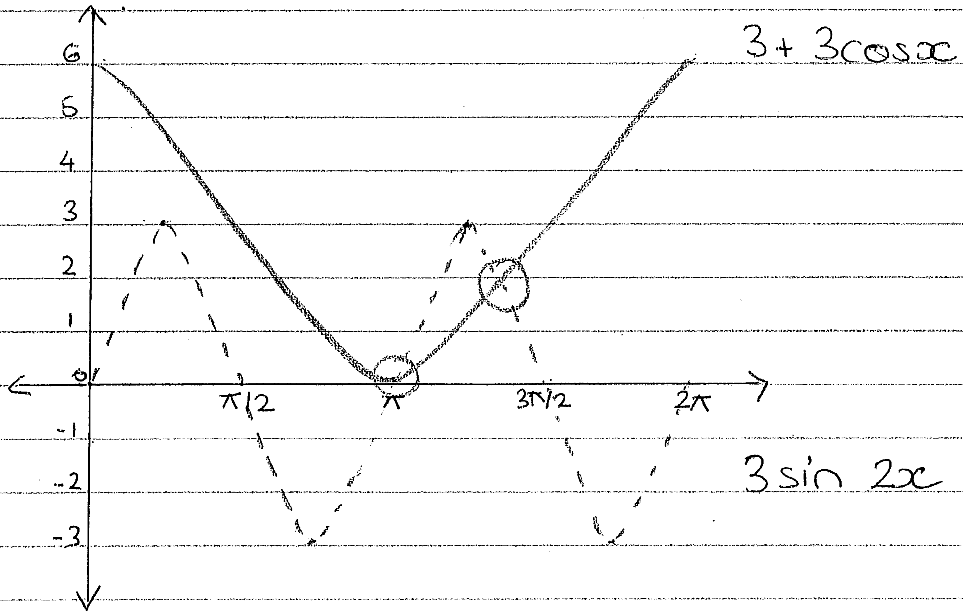
$0 = 3x(x - 2)$

$3x = 0\ ,\quad x - 2 = 0$

$x = 0\ ,\quad x = 2$

b) i)

iii)

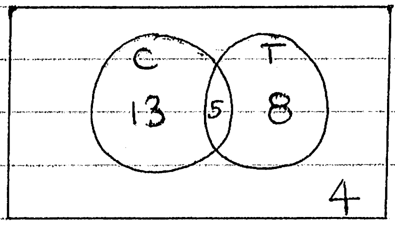


ii) $y = 3\sin 2x$. amplitude $= 3$

period $= \dfrac{2\pi}{2} = \pi$

iv) There are 2 solutions.

c) class.



$P(\text{neither}) = 4/30$
$= 2/15$

d)

x	0	10	20	30	40
$f(x)$	20	14	12	7	8

$h = \dfrac{40-0}{4} = \dfrac{40}{4} = 10$

$$\int_0^{40} f(x)\,dx \doteq 10/3\,[(20+8) + 4(14+7) + 2(12)]$$
$$\doteq 10/3\,[28 + 4(21) + 2(12)]$$
$$= 453\ 1/3\ m^2$$

e) $f'(x) = 3x^2 - 1$ P.t. point $(4, 1)$

$f(x) = \int f'(x)$
$= \int 3x^2 - 1$
$= x^3 - x + C$

$f(4) = 1 \quad \therefore \quad 1 = 4^3 - 4 + C$
$1 = 64 - 4 + C$
$1 = 60 + C$
$C = -59$

$\therefore\ f(x) = x^3 - x - 59$

Question 4

a) i) P(1st prize) $= 5/100$
$= 1/20$

ii) P(2nd prize) = P(1st & 2nd prize) + P(not 1st & 2nd prize)
$= (5/100 \times 4/99) + (95/100 \times 5/99)$
$= 1/20.$

b) $y = x^4 - 4x^3$

$y' = 4x^3 - 12x^2$ $\qquad$ $y'' = 12x^2 - 24x$

i) SP @ $y' = 0$
$0 = 4x^3 - 12x^2$
$0 = 4x^2(x-3)$
$4x^2 = 0$, $x - 3 = 0$
$x = 0$, $x = 3$

when $x = 0$, $y = 0$
$\therefore$ SP @ $(0, 0)$
when $x = 3$, $y = 3^4 - 4 \times 3^3$
$= -27$
$\therefore$ SP @ $(3, -27)$

ii) $y'' = 0$ helps determine nature of st. pts.

$y''(0) = 0 - 0$
$= 0$
- possible pt of inflexion

$y''(3) = 12 \times 3^2 - 24 \times 3$
$= 36$
$\therefore (3, -27)$ is a local minimu

check y' for sign of derivative.

x	-1	0	1
y'	-16	0	-8

$\therefore (0,0)$ is a pt of inflexion.

iii) $y'' > 0$ for concave up; $\therefore$ $12x^2 - 24x > 0$
$x^2 - 2x > 0$
$x(x-2) > 0$
$\therefore$ $x > 2$, $x < 0$
$\therefore$ the curve is concave up for $x > 2$, $x < 0$.

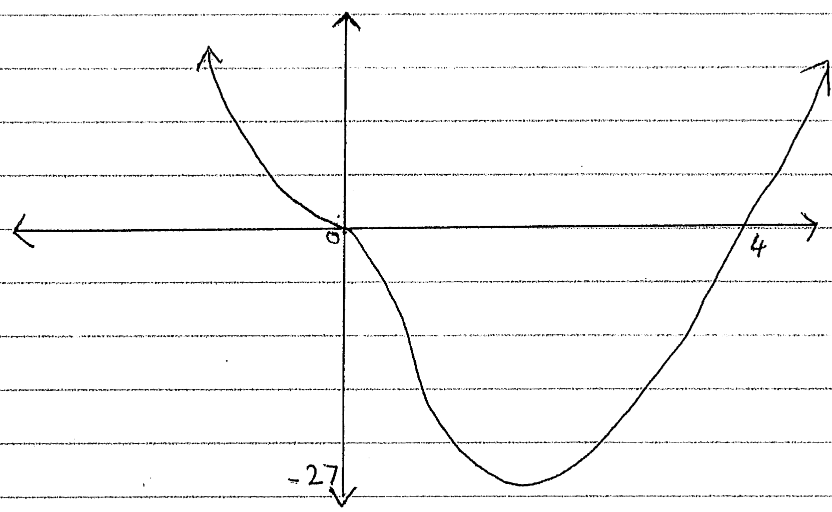
iv) $f'(x) < 0$ for curve decreasing.
$4x^3 - 12x^2 < 0$
$x^2(x-3) < 0$
$x^2 < 0$, $x < 3$.

$\therefore$ the curve is decreasing for $x < 3$, $x \neq 0$

c) $P(E) = 1/x$

$P(\text{not } E) = 1 - 1/x$
$= \dfrac{x-1}{x}$

Q4b) v)

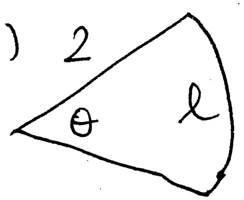


Solution to Section (c)

Question (5).

(a)

(i) 2



$A = \frac{1}{2} r^2 \theta$

$1.6 = 2\theta$

$\therefore \theta = 0.8$ *

i.e $0.8^c = \frac{180}{\pi} \times 0.8$

$\doteqdot 45^\circ 50'$ *

[2]

(ii) $l = r\theta$

$= 2 \times 0.8$

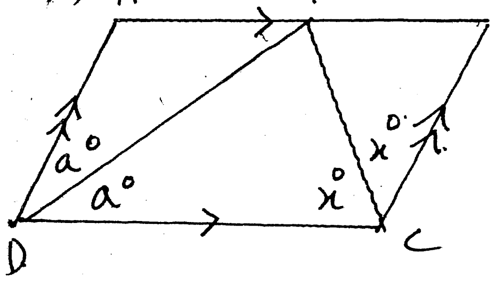
$= 1.6$ *

$\therefore p = l + 2r = 5.6\text{ m}$

$= 560\text{ cm}.$ *

[2]

(b)



(i) In parm^t ABCD.

$\angle ADC + \angle BCD = 180^\circ$

(co-interior $\angle$s are supplementary, AD ∥ BC)

$\therefore 2a + 2x = 180$

$a + x = 90^\circ.$

$\angle DPC = 180^\circ - (a+x)$

(Angle sum of $\triangle DPC$)

i.e $\angle DPC = 180^\circ - 90^\circ$

$= 90^\circ$ [2]

(ii) In $\triangle APD$. [2]

$\angle PDC = \angle APD = a^\circ$

(alternate $\angle$s, AP ∥ DC)

$\therefore \angle ADP = \angle APD = a^\circ$

i.e $\triangle ADP$ is isosceles.

(iii) Similary (from (ii))

- BP = BC. ($\triangle BPC$ is isosceles)

and AD = AP (proven).

- but AD = BC

(opposite sides of a parm equal). [2]

$\therefore$ AB

= AP + PB

= AD + BC.

= 2BC.

(c)

(i) $\int_0^8 f(x)\,dx$

$= \frac{1}{4}(\pi \times 2^2) + 2^2 + \frac{1}{2}(2) \times 2 - \frac{1}{2}(2) \times 2$ [2]

$= \pi + 4 \quad (7.142)$

(ii) At $x = 4$ f is not differentiable. [1]

(d) At $x = 5$ [2]

$f'(5) = 0$

$f'(5-\varepsilon) > 0$ (for small positive ε)

$f'(5+\varepsilon) < 0.$

$\therefore$ By the 1st derivative test f(x) has a local maximum at (5, 0).

Question (6).

(a) $y = -x^2 + 7x - 6$

$y = x + 2$

$-x^2 + 7x - 6 = x + 2$.

(i) $x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$\therefore\ x = 2,\ y = 4$; $x = 4,\ y = 6$

$(2, 4)\quad (4, 6)$ [2]

i.e $A = 2$ and $B = 4$.

(ii) $A = \int_2^4 \left[(-x^2+7x-6) - x - 2\right] dx$

$= \int_2^4 (-x^2 + 6x - 8)\, dx$

$= \left[-\frac{x^3}{3} + 3x^2 - 8x\right]_2^4$

$= \left[-\frac{64}{3} + 48 - 32\right] - \left[-\frac{8}{3} + 12 - 16\right]$

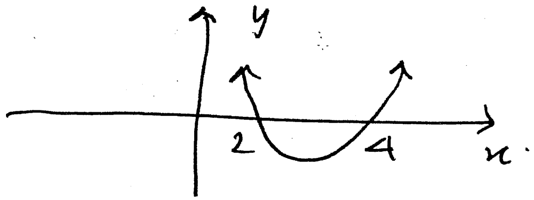
$= -\frac{56}{3} + 16 + 4$

$= 4/3$. [2]

(iii) $-x^2 + 6x - 8 > 0$

$x^2 - 6x + 8 < 0$

$(x-4)(x-2) < 0$



$2 < x < 4$ [2]

(b) $y = -2x^2 + 6x + 3$

$A(2, 7)$

$\frac{dy}{dx} = -4x + 6$

$\left.\frac{dy}{dx}\right|_{x=2} = -2$

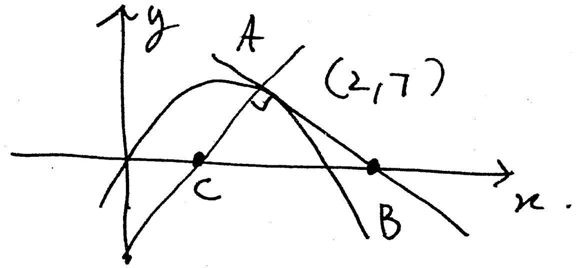
$\therefore\ y - 7 = -2(x - 2)$

$2x + y - 11 = 0$ [tgt]

$y - 7 = \frac{1}{2}(x - 2)$

$2y - 14 = x - 2$

$\therefore\ x + 2y + 12 = 0$ (normal) [4]



$2x = 11 \qquad x = \frac{11}{2}$ [2]

$x = -12$

$B\left(\frac{11}{2}, 0\right) \quad C(-12, 0)$

$\therefore$ Area $= \frac{1}{2} \times 17\frac{1}{2} \times 7$

(c) Let $BC = x$.

(i) $\frac{7}{7+x} = \frac{\cancel{6}}{\cancel{30}}\ \frac{3}{10}$

$70 = 21 + 3x$

$9 = 3x$

$x = 49/3$. [2]

(ii) $BF : DE$

$7 : \frac{49}{3}$

$21 : 49$

$3 : 7$ [2]